

E-B Mixing in T-violating Superconductors

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February 1, 2008

Abstract

We analyze time-reversal-violating processes of the p-wave superconductor. The Landau-Ginzburg effective action has an induced T-violating term of electromagnetic potentials which resembles the Chern-Simons term and causes mixing between the electric and magnetic fields. Several T-violating electromagnetic phenomena caused by this term, such as an unusual Meissner effect, the Hall effect without a magnetic field, and Faraday rotation without a magnetic field are investigated.

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Time reversal symmetry holds in ordinary matter in macroscopic systems because the electromagnetic interaction, which is the origin of all forces in these regions, preserves the time-reversal symmetry. The symmetry is broken explicitly by an external magnetic field or spontaneously by condensation of a T-odd order parameter. Interesting physical phenomena would then occur. The Chern-Simons term is a T-odd bilinear form of electromagnetic potentials and has one derivative. Hence this term is the lowest dimensional gauge invariant object and plays important roles in the low-energy and long-distance physics of the T-violating system[1, 2, 3, 4, 5, 6].

The Chern-Simons term plays an important role in the quantum Hall effect [5, 6][7, 8, 9, 10], where the time-reversal symmetry is broken explicitly by the external magnetic field. In this paper we discuss time-reversal-violating processes of the p-wave superconductor[11][12] in which time-reversal symmetry is broken spontaneously. We assume that pairs with the angular momentum of one along the z-axis condense homogeneously as in the A-phase of ^3He [13], and we also call this phase as the A-phase of the p-wave superconductor [14][15]. We show, in this letter, several unusual electromagnetic phenomena, such as mixing between the electric and magnetic fields (E-B mixing) in the superconductor.

Our discussion below may be valid for other T-violating superconductors, because the discussion does not depend on the detail forms of order parameters, but only on the fact that Landau-Ginzburg action has a T-odd Chern-Simons-like term.

The long-range effective action for gauge fields in the A-phase of the ^3He like superconductor at the Fermion 1-loop level is given by [16] [17][18]

$$S_{\text{eff}}^{(f)} = \int d^D x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m_B^2}{2} \left(\frac{1}{c^2} A_0^2 - A_i^2 \right) + \frac{\sigma}{2} e_{ij} (A_0 \partial_i A_j + A_i \partial_j A_0) \right] + \mathcal{O}(e^3). \quad (1)$$

D represents space-time dimension of the system and we consider cases of both 2+1 and 3+1 dimensions. The second term in eq.(1) represents the Meissner effect and $1/m_B$ shows the penetration depth of the magnetic field if the last term is ignored. Parameter c is the speed of sound in superconductors. The third term is odd under the time-reversal transformation and resembles the Chern-Simons term, but is not totally antisymmetric about space-time indices ($e_{ij} = \varepsilon_{ij}$ in the 2+1 dimensions, and $e_{ij} = \varepsilon_{ij3}$ in the 3+1 dimensions). Parameters (m_B, c, σ) vary depending on the spatial dimension. For the A phase, they are given in Table I.

We study the characteristic features of T-violating systems based on the above action and ignore the order-parameter-dependent terms. Order-parameter-dependent terms depend on the model used and their properties have been given in the literature.

We hence ignore them and discuss general time-reversal- violating properties derived from eq.(1) in the present paper.

To elucidate the E-B mixing effects, we first study the $^3\text{He-A}$ - like superconductor in a static electric field. Usually, nothing happens except the screening of the electric field in conventional superconductors, but in the present case, a magnetic field and an electric current are induced in the superconductor.

Suppose there is a superconductor in the $0 < x$ region and there is a static background electric field $E_x^{(b)}$ directed along the x-axis in the $x < 0$ region. We call the $x < 0$ region as region I, and the $0 < x$ region as region II. Thus, there is a boundary at the y-z plane (y-axis) in the 3+1-dimensional space-time (2+1 dimensional space-time)(Fig.1).

The action in both regions is

$$\begin{aligned} S_I &= \int d^D x \left[-\frac{1}{4} F_{I\mu\nu} F_I^{\mu\nu} - \frac{1}{2} F_{I\mu\nu} F_I^{(b)\mu\nu} + \frac{1}{2} E_x^{(b)2} \right] \\ F_I^{\mu\nu} &= \partial^\mu A_I^\nu - \partial^\nu A_I^\mu \\ F_I^{(b)10} &= E_x^{(b)}, \text{ otherwise } 0 \\ S_{II} &= S_{\text{eff}}^{(f)}. \end{aligned} \quad (2)$$

From the variational principle, we can derive the equations of motion in both regions under boundary conditions at $\partial I (= -\partial II)$, which is the surface of region I (region II, but opposite orientation). The equations of motion are written as

$$\begin{aligned} &\text{in the region I ; } \partial_\mu F_I^{\mu\nu} = 0, \\ &\text{in the region II ; } \partial_\mu F_{II}^{\mu\nu} = j_\nu^{II}, \end{aligned} \quad (3)$$

j_μ^{II} 's are currents and their forms are

$$j_0^{II} = -\frac{m_B^2}{c^2} A_0^{II} - \sigma e_{ij} \nabla_i A_j^{II} + \frac{\sigma}{2} A_y^{II} \delta(x) \quad (4)$$

$$j_i^{II} = -m_B^2 A_i^{II} + \sigma e_{ij} \nabla_j A_0^{II} + \frac{\sigma}{2} e_{1i} A_0^{II} \delta(x) . \quad (5)$$

The first term on the right hand side of eq.(5) shows the London current, the second is the Hall current which flows perpendicular to the electric field, and the third is an edge current which exists only on the boundary surface. The latter two terms arise from the induced T-violating term and its surface term at the boundary. The boundary conditions are written as

$$\begin{aligned} &\int_{\partial I} d\sigma_\mu [-F_I^{\mu\nu} - F_I^{(b)\mu\nu} + F_{II}^{\mu\nu}] \delta A_\nu \\ & - \frac{\sigma}{2} \int_{\partial I} d\sigma_i e^{ij} (A_0^{II} \delta A_j^{II} - A_j^{II} \delta A_0^{II}) = 0, \end{aligned} \quad (6)$$

where $d\sigma_\mu$ is a unit area vector of surface ∂I .

We solve gauge fields of static configurations, which have only x -dependence because of the spatial symmetry of the system. We assume that in region I there is a constant electric field. Hence, A_μ^{I} should be constant and are connected continuously with gauge fields in region II at the boundary. Equations of motion in region II reduce to

$$\begin{aligned}\frac{d^2}{dx^2}A_0^{\text{II}}(x) &= \frac{m_{\text{B}}^2}{c^2}A_0^{\text{II}}(x) + \sigma\frac{d}{dx}A_y^{\text{II}}(x) \\ \frac{d^2}{dx^2}A_i^{\text{II}}(x) &= m_{\text{B}}^2A_i^{\text{II}}(x) + \sigma e_{1i}\frac{d}{dx}A_0^{\text{II}}(x).\end{aligned}\quad (7)$$

A_x^{II} and A_z^{II} (only in the 3+1 dimensional case) satisfy the ordinary London equation, but A_0^{II} and A_y^{II} are influenced by the T-violating term and they mix. Since $c \ll 1$ (Table I), eigenvalues of the mixing equation are

$$q_+ \simeq \frac{m_{\text{B}}}{c}, \quad q_- \simeq m_{\text{B}}, \quad (8)$$

and corresponding eigenvectors are written as

$$\vec{v}_+ = N_+ \begin{pmatrix} q_+^2 - m_{\text{B}}^2 \\ -\sigma q_+ \end{pmatrix}, \quad \vec{v}_- = N_- \begin{pmatrix} -\sigma q_- \\ q_-^2 - \frac{m_{\text{B}}^2}{c^2} \end{pmatrix} \quad (9)$$

Both N_+ and N_- are normalization constants for each eigenvector. Using these eigenvalues and eigenvectors, A_0 and A_y are written as

$$\begin{pmatrix} A_0 \\ A_y \end{pmatrix} = C_0 \vec{v}_+ e^{-q_+ x} + C_y \vec{v}_- e^{-q_- x}. \quad (10)$$

Amplitudes C_0 and C_y are decided by boundary conditions (6) at the surface, $x = 0$, which reduce to

$$\begin{aligned}\lim_{x \rightarrow +0} [-\frac{d}{dx}A_0^{\text{II}}(x) + \frac{\sigma}{2}A_y^{\text{II}}(x)] &= E_x^{(\text{b})} \\ \lim_{x \rightarrow +0} [\frac{d}{dx}A_y^{\text{II}}(x) - \frac{\sigma}{2}A_0^{\text{II}}(x)] &= 0.\end{aligned}\quad (11)$$

As a result of E-B mixing a magnetic field is induced in region II (*i.e.*, $B_z^{\text{II}}(x) = \frac{d}{dx}A_y^{\text{II}}(x)$). By taking the leading terms of the power expansion of c , the solution is written as

$$B_z^{\text{II}}(x) \simeq E_x^{(\text{b})} \frac{c\sigma}{m_{\text{B}}} [e^{-m_{\text{B}}x/c} - \frac{1}{2}e^{-m_{\text{B}}x}]. \quad (12)$$

According to eq.(12), discontinuity of the magnetic field arises at the boundary, and its magnitude is $\mathcal{O}(\frac{c\sigma}{m_B}E_x^{(b)})$. This originates from an extra term, $\lim_{x \rightarrow +0} \frac{\sigma}{2}A_0$, in the boundary condition for the magnetic field in eqs.(11). Spatial dependence of the field is specified by two length scales, m_B^{-1} and $(m_B/c)^{-1}$, and the field has the extremum value before it tends to zero at $x \rightarrow \infty$. Its behavior is shown in Fig.2. Using the Maxwell equation, we obtain an electric current directed along the y-axis, which is written as

$$j_y^{\text{II}}(x) = (\vec{\nabla} \times \vec{B}^{\text{II}}(x))_y \quad (13)$$

$$\simeq -\sigma E_x^{(b)} [-e^{-m_B x/c} + \frac{c}{2}e^{-m_B x} - \frac{c}{2m_B}\delta(x)].$$

We are able to derive the same result from eq.(5). As mentioned previously, the current consists of three components, the London current, the Hall current, and the edge current. The Hall current is written as

$$j_{y(\text{H})}^{\text{II}}(x) = \sigma \nabla_x A_0^{\text{II}}(x) \quad (14)$$

$$\simeq -\sigma E_x^{(b)} [e^{-m_B x/c} - \frac{1}{2} \frac{c^3 \sigma^2}{m_B^2} e^{-m_B x}].$$

In general, the Hall current occurs in the background magnetic field, but in the present case, Hall current is generated not by the background magnetic field, but by the induced magnetic field due to E-B mixing.

At the cylinder surface boundary, a magnetic moment is induced by the above Hall current. We assign a as the radius of the cylinder, and it takes a value much larger than the length $1/m_B$. In this case, we choose the cylinder coordinates. Corresponding to $j_y^{\text{II}}(x)$, an angular current $j_\theta^{\text{II}}(r)$ exists and they have the relation $j_\theta^{\text{II}}(r) \simeq -j_y^{\text{II}}(a-r)$ when a is large. The magnetic moment M_z^{ind} induced by j_θ^{II} is written as

$$M_z^{\text{ind}} = e \int d\theta dr r^2 j_\theta \simeq e a^2 \frac{2\pi c \sigma}{m_B} E_x^{(b)}. \quad (15)$$

This induced magnetic moment[19] is proportional to the external electric field. These results eqs.(12),(13), and (15) originate from the T-violating term, because they vanish when $\sigma = 0$.

Instead of the background electric field $E_x^{(b)}$, we consider a background magnetic field $B_z^{(b)}$ directed along the z -axis. Except for boundary conditions (11), this calculation is the same as in the previous case. We obtain the behavior of the magnetic field in region II as

$$B_z^{\text{II}}(x) \simeq B_z^{(b)} [-\frac{c\sigma^2}{2m_B^2} e^{-m_B x/c} + e^{-m_B x}]. \quad (16)$$

This shows the Meissner effect in the A-phase of the p-wave superconductor. It is unusual because the magnetic field has two damping modes. However this effect is almost invisible because one of the mass eigenmodes m_B/c is much larger than the other one m_B , and the coefficient of $e^{-m_B x/c}$ is much smaller than that of $e^{-m_B x}$. The electric current

$$j_y^{\text{II}}(x) \simeq B_z^{(\text{b})} \left[\frac{\sigma^2}{2m_B} e^{-m_B x/c} + m_B e^{-m_B x} - \frac{c\sigma^2}{4m_B^2} \delta(x) \right] \quad (17)$$

also occurs in the present case and is affected by the T-odd term. However the ordinary supercurrent also occurs in superconductors without T-violation (*i.e.*, $\sigma = 0$) under the magnetic field. It is difficult to observe the T-violating effect experimentally in eq.(17) compared with the previous case. This is also the same for the induced magnetic moment.

Finally, we show that the T-odd term in action (1) causes T-violating light scattering at the surface of the superconductor. We consider the reflection of polarized light injected from the outside of the superconductor along the x direction (Fig. 1) and calculate the rotation of the polarization plane in the reflected wave. This effect, which we call Faraday rotation here, has already been investigated by Wen and Zee[20][21] [6] in a T-violating superconductor. They showed that a T-odd term in the Landau-Ginzburg action, e.g., $\epsilon_{ij} A_i \partial_0 A_j$ causes this effect. In this letter, we show that this effect also occurs with the T-odd term in eq.(1).

We study time-dependent solutions of eqs.(3). We obtain a condition for the gauge fields in the superconductor by taking the divergence of both sides of eq.(3) (omitting the suffix)

$$\partial^\mu j_\mu = 0. \quad (18)$$

This condition is nontrivial in the system with broken gauge invariance , but is trivial in the gauge invariant system. Using this condition, eq.(3) for the superconductor become as follows,

$$(\partial_0^2 - c^2 \partial_x^2 + m_B^2) A_0 = -c^2 \frac{\sigma}{m_B^2} (\partial_0^2 + m_B^2) \partial_x A_y \quad (19)$$

$$(\partial_0^2 - c^2 \partial_x^2 + m_B^2) A_x = -c^2 \frac{\sigma}{m_B^2} \partial_0 \partial_x^2 A_y \quad (20)$$

$$(\partial_0^2 - \partial_x^2 + m_B^2) A_y = -\sigma \partial_x A_0 \quad (21)$$

$$(\partial_0^2 - \partial_x^2 + m_B^2) A_z = 0. \quad (22)$$

According to eqs.(19) and eq.(21), A_0 and A_y mix, and their propagation is specified by two momentum eigenmodes $q_+ \simeq \frac{(k_0^2 - m_B^2)^{1/2}}{c}$ and $q_- \simeq (k_0^2 - m_B^2)^{1/2}$, when $c \ll 1$,

where k_0 is the frequency of light. Equation (20) shows that A_x is caused by A_y which would be considered as a source of force vibration. A_z does not mix with the others. Condition (18) suggests that the solution of eq.(20) does not have the freedom to add kernel, which is the same solution as when the r.h.s. of eq.(20) is zero. The gauge fields in region II should be written as

$$\begin{aligned} A_\mu &= c_{\mu+} e^{-ik_0 t + i q_+ x} + c_{\mu-} e^{-ik_0 t + i q_- x} ; \mu = 0, 1, 2 \\ A_z &= c_z e^{-ik_0 t + i \sqrt{k_0^2 - m_B^2} x}. \end{aligned} \quad (23)$$

The coefficients $c_{\mu+,-}$, and c_z will be chosen to satisfy the equations of motion, condition (18), and the boundary conditions. The gauge fields in region I are written as

$$A_\mu = a_\mu e^{-ik_0(t-x)} + b_\mu e^{-ik_0(t+x)}. \quad (24)$$

The first term on the r.h.s. of eq.(24) indicates the incident wave and the second indicates the reflected wave. Boundary conditions at $x = 0$ require that the gauge fields connect continuously and that their derivatives obey eq.(6). In this case, there are no background fields in the region I (*i.e.*, $E^{(b)} = B^{(b)} = 0$). Using the boundary conditions, we determine the ratios of $c_{\mu+,-}$, c_z and b_μ . We set the linearly polarized incident wave along the y-direction (*i.e.*, $a_y = 1, a_0 = a_x = a_z = 0$). b_μ for $k_0 \ll m_B$ are written as

$$b_0 \simeq \frac{2ik_0 c^2 \sigma}{m_B^2}, \quad (25)$$

and for $k_0 \simeq m_B$, are written as

$$b_0 \simeq \frac{4k_0}{\sigma}, c\sigma(m_B^2 - k_0^2)^{1/2}, \quad (26)$$

In both cases, $b_z = 0$. The effect of the T-violation is seen in b_0 and b_x and the rotation of polarization occurs. When $k_0 \simeq m_B$ a resonance occurs, but near the resonance, the damping effect of the vibration that is neglected here should be taken into account. Furthermore, we should note that light absorption of the superconductor occurs in the region of energy larger than the gap energy $|\Delta|$ because quasi-particle production occurs in such an energy region. Therefore in the case of $|\Delta| \ll m_B$, the resonance is not detectable. However in some superconductors the parameters would satisfy $m_B < |\Delta|$. The energy dependence of the absolute values of these amplitudes are given in Fig.3 in the region $0 < k_0 < m_B$ for heavy-fermion superconductors and high- T_c superconductors [11, 12][14, 15]. T-violating amplitude has a significant magnitude.

In this letter, we showed some unusual electromagnetic phenomena in the A-phase of a p-wave superconductor. This system violates T spontaneously, so it has an induced Chern-Simons-like T-violating term in the Landau-Ginzburg effective action. Owing to this term, mixing between the electric and magnetic fields occurs. In the system with an external electric field, a magnetic field and an electric current are induced. The current includes a Hall current. This suggests that a Hall effect occurs without the external magnetic field. The current gives the magnetic moment of the system. In the system with a magnetic field, an unusual Meissner effect occurs and the magnetic field in the superconductor obeys two damping modes. We have shown also that T-violating light scattering, such as Faraday rotation, occurs at the surface of the p-wave superconductor A-phase. Such effects that were studied here are generated by the T-odd Chern-Simons-like term. They should show characteristic features of T-violating systems and may also occur for other T-violating superconductors.

The authors are grateful to Dr. N. Maeda for useful discussions. This work was partially supported by a special Grant-in-Aid for the Promotion of Education and Science in Hokkaido University provided by the Ministry of Education, Science, Sports, and Culture, Grants-in-Aid for Scientific Research (07640522), for Scientific Research on Priority area (Physics of CP violation), and for International Science Research (Joint Research 07044048) from the Ministry of Education, Science, Sports and Culture, Japan.

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Captions

Table.I. Parameters in A phase. m_e, ρ , and ϵ_F are mass, number density, and the Fermi energy of electrons.

Fig.1. This Figure shows the situation of our calculation. there is a background electric field $E_x^{(b)}$ directed to the x-axis in the region I, and the $^3\text{He-A}$ like superconductor (represented by shadow lines) is in the region II.

Fig.2. The induced magnetic field $B_z^{\text{II}}(x)$ in the superconductor under a background electric field $E_x^{(b)}$ directed to x-axis.

Fig.3. Energy dependence of the logalizm of $|b_0|$, $|b_x|$, and $|b_y|$ when $a_y = 1$ and others are 0. Parameters c, σ, m_B are chosen as typical values in High- T_c superconductors (solid lines) and heavy fermion superconductors (dashed lines).

Table.I

parameters	2+1 dimension	3+1 dimension
m_B^2	$\frac{\rho e^2}{m_e}$	$\frac{\rho e^2}{m_e}$
c^2	$\frac{\rho}{m_e^2}$	$\frac{2\epsilon_F}{3m_e}$
σ	$\frac{e^2}{4\pi}$	$\frac{\rho e^2}{8m_e\epsilon_F}$